High School Proficiency Assessment (HSPA)

A Mathematics Handbook: Open-Ended Questions

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HIGH SCHOOL PROFICIENCY ASSESSMENT (HSPA)

A MATHEMATICS HANDBOOK: OPEN-ENDED QUESTIONS

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HIGH SCHOOL PROFICIENCY ASSESSMENT (HSPA)
PROGRAM DESCRIPTION

In 1975, the New Jersey Legislature passed the Public School Education Act "to provide to all children in New Jersey, regardless of socioeconomic status or geographic location, the educational opportunity which will prepare them to function politically, economically and socially in a democratic society." An amendment to that act was signed in 1976, which established uniform standards of minimum achievement in basic communication and computation skills. This amendment is the legal basis for the use of a test as a graduation requirement in the State of New Jersey.

Beginning in 1981–82, ninth-grade students were required to pass the Minimum Basic Skills Test (Reading and Mathematics) as one of the requirements for a high school diploma. Students who did not pass both parts of the test had to be retested on those parts not passed.

In 1983, a more difficult test in Reading, Mathematics, and Writing was adopted, the Grade 9 High School Proficiency Test (HSPT9), to measure the basic skills achievements of ninth-grade students. The first due-notice administration of the HSPT9 occurred in 1983–84; the first time the test was administered as a graduation requirement was 1985–86.

In 1988, the New Jersey Legislature passed a law which moved the High School Proficiency Test from the ninth grade to the eleventh grade. The Grade 11 High School Proficiency Test (HSPT11) was a rigorous test of essential skills in Reading, Mathematics, and Writing. It served as a graduation requirement for all public school students in New Jersey who entered the ninth grade on or after September 1, 1991. Three years of due-notice testing were conducted to allow school districts time to modify curricula and prepare students for the graduation test.

In 1996, the New Jersey State Board of Education adopted Core Curriculum Content Standards to describe what all students should know and be able to do at the end of fourth grade, eighth grade, and upon completion of a New Jersey public school education. The Core Curriculum Content Standards delineate New Jersey's expectations for student learning. All New Jersey school districts are required to organize instruction and design curricula so that virtually all students achieve the new content standards. The Core Curriculum Content Standards ultimately define the state's high school graduation requirements and its testing program to measure benchmark achievements toward those requirements in grades 4, 8, and 11.

The Elementary School Proficiency Assessment (ESPA), which was administered to fourth- and fifth-graders, was designed from its inception in 1997 to align with the content standards, as is the New Jersey Assessment of Skills and Knowledge (NJASK), which replaced the ESPA. The Grade Eight Proficiency Assessment (GEPA), which replaced the Grade 8 Early Warning Test (EWT) administered to eighth-graders from 1991 to 1996, is additionally aligned with the content standards. The GEPA should be used for placement purposes and program planning for appropriate instruction to enable students to ultimately pass the state's graduation test. The High School Proficiency Assessment (HSPA), which is also aligned with the content standards and has replaced the HSPT11 as the state's graduation test, was field tested for a three-year period. The HSPA was administered to eleventh-graders as a graduation test for the first time in March 2002.

HSPA MATHEMATICS & OPEN-ENDED QUESTIONS

The mathematics section of the High School Proficiency Assessment measures a student's ability to solve problems by applying mathematical concepts. The areas tested are as follows: Number and Numerical Operations; Geometry and Measurement; Patterns and Algebra; and Data Analysis, Probability, and Discrete Mathematics.

The mathematics section of the test consists of four parts containing multiple-choice questions and open-ended questions. Each section contains 10 multiple-choice questions and 2 open-ended questions for a total of 40 multiple-choice and 8 open-ended questions. It is expected that students will take approximately 1 to 2 minutes to answer each multiple-choice question and approximately 10 minutes to answer each open-ended question.

Responses to the open-ended questions must be made in the area provided in the answer folder. Specific directions with each question will refer the student to the page in the answer folder where the response is to be written. For each of these questions, a student must provide enough explanation so that the scorer can understand the solution. The student's response will be scored on the correctness of the method as well as the accuracy of the answer. No credit will be given for anything written in the test booklet. Responses must be in English in order to be scored.

The open-ended questions will be hand scored on a scale from 0 to 3. The general scoring guide on page 3 was created to help trained readers score open-ended questions consistently. Each question on the HSPA has its own scoring rubric, which is based upon the general scoring guide.

The students are provided with a Mathematics Reference Sheet, as shown on page 7. The reference sheet contains a ruler, geometric shapes, formulas, and other information the student may find useful as he/she takes the test. The student is also provided with a calculator to help him/her solve problems.

Scoring Guide for Mathematics Open-Ended (OE) Questions
(Generic Rubric)

3-Point Response

The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.

2-Point Response

The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

1-Point Response

The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0-Point Response

The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution, or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

The above scoring guide is used to develop specific scoring rubrics for each of the open-ended (OE) questions that appear on the New Jersey statewide assessments in Mathematics. These scoring rubrics provide the criteria for evaluating and scoring student performance and are developed by a committee of mathematicians and teachers. Rubrics ensure that there is consistency, fairness, and accuracy in scoring open-ended questions.

OPEN-ENDED SCORING FOR MATH

Scoring with the Criteria

Each New Jersey high school student open-ended response for math is scored by two independent readers at Measurement Incorporated (MI), the HSPA test contractor.

To score the high school open-ended responses for math, MI selects approximately 100 of its most experienced readers, all of whom must possess a four-year college degree. Each group of readers is trained on and scores only two items. All readers, regardless of experience, are required to participate in an intensive three-day training period for the items they will score. Only readers who meet the 80% perfect agreement standard on at least two training sets qualify to score New Jersey math items. By the end of training, the readers have "internalized" the defined criteria at each of the four score points of the rubrics for each item by practice scoring and discussing sample student responses.

Scoring Personnel and Procedures

Current procedures for scoring student open-ended items on the HSPA are consistent with those used by New Jersey since the inception of the statewide assessment. The scoring of all open-ended items on the HSPA is monitored by trained, experienced personnel. Many individuals are responsible for ensuring the success of scoring any large-scale assessment. Key to the process of scoring New Jersey's high school responses accurately and reliably are MI's senior project manager, the chief readers (scoring directors), team leaders, readers, and clerical aides.

MI's senior project manager works closely with the department throughout the handscoring process. The senior project manager participates in all rangefinding and training paper selection activities prior to the onset of team leader and reader training. The senior project manager directs the activities of the chief reader and oversees all aspects of the project including monitoring reader performance (reader reliability and production rates), directing retraining efforts, and supervising the capture of scoring data.

The chief reader participates in rangefinding and training paper selection along with MI's senior project manager and the department's mathematics assessment specialists. Additionally, the chief reader annotates the anchor papers that, along with the scoring criteria, make up the Scoring Guide. He or she also trains the team leaders who will subsequently assist in reader training. It is the responsibility of the chief reader to introduce the open-ended items, rubrics, and sample responses; to conduct the majority of the training sessions (some training sets are discussed in teams); and to ensure that readers score reliably and consistently throughout the scoring process. The chief reader supervises the team leaders, directs all scoring and validity procedures, reads and interprets reader quality control reports, and conducts all retraining activities. Additionally, the chief reader assigns all nonscorable codes and does resolution readings.

Each team leader is responsible for small-group training sessions with the eight to ten readers who constitute his/her team. Under the supervision of the chief reader, some training sets are

discussed in teams to encourage more questions from individual readers and to allow team leaders to get a clearer picture of the level of understanding of each team member.

Team leaders rely heavily upon periodic individual and small-group retraining to correct reader drift—that is, scoring that is not in accord with the criteria. They spot-check reader scoring packets throughout the project and counsel readers who have a higher than acceptable discrepancy rate. An item is considered discrepant if two independent readers assign non-adjacent scores to the same response (e.g., one reader assigns a "3," the second reader a "1." These responses require a resolution reading by the chief reader or a team leader.) Additionally, team leaders meet daily as a group with the chief reader and discuss any scoring differences to guard against team "drift."

Once trained, the readers' primary task is to score accurately all high school math open-ended items. To accomplish this task, clerical aides distribute scoring packets containing 30 responses and score sheets to each team. The readers, upon taking a packet, record their reader number, team designation, and the date on the scoring packet. The first reader of the packet then codes his/her reader number on the Reader 1 score sheet and proceeds to score all the papers in that packet. Student identification numbers on the score sheet are checked carefully against the numbers on the student response document to make sure that they are in agreement. If there is an error, the packet is flagged (marked with a sticker) for the aide to check. If the aide is unable to correct the error, the packet is given to the chief reader. After all papers in a given packet have been scored once, the aide collects the scored packet, places the first reader score sheet in a bin for scanning, and distributes the packet to a different team for a second reading. The second reader follows the same procedures as the first reader, but uses the Reader 2 score sheet. At no time does the second reader have access to the first reader's score.

Readers are also responsible for recognizing and flagging nonscorable responses (fragment, off-topic, not English, no response) and "alert" papers (e.g., suspicion of child abuse) so that these papers can be handled in the correct manner. Alert papers are scored, but then forwarded to the chief reader for review. If the chief reader agrees that the student's own words specifically state that a situation qualifies as an alert or reflect a potential risk situation for a child, the paper is copied and sent to the department for documentation and follow-up with district authorities. The Office of Evaluation and Assessment in the Department of Education brings these alerts to the attention of school district personnel. Alert papers are flagged if they reflect potential abuse, emotional or psychological difficulty, or possible plagiarism.

The clerical aides play an important role in maintaining the paper flow throughout the scoring process. They are responsible for keeping enough packets in the scoring room to keep the readers busy. This includes distributing packets for first readings and also directing packets that have one reading completed to different teams for second readings. Once packets have been read twice, the aides take them to the warehouse for filing. In addition, the aides collect completed score sheets and forward them to the scanning room, where scores are scanned into the database. If any packets produce resolution readings, the aide retrieves them from the warehouse and gives them to the chief reader for adjudication.

DESCRIPTION OF THIS MANUAL

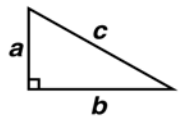
This manual contains four open-ended items, one from each of the Core Curriculum Content Standards. The question, sample solution, and item-specific scoring rubric are provided for each item. Three exemplar papers for each of the four score points are included for each of the open-ended items. These sample responses, which are grouped by score point, represent the range of approaches that high school students take with this open-ended item in mathematics. Each response is annotated according to the score point criteria.

The responses selected to appear in this handbook were written by high school students. The responses appear as the students wrote them; no corrections have been made other than the deletion of specific names that may have appeared to identify the student or the student's school district.

HIGH SCHOOL PROFICIENCY ASSESSMENT
MATHEMATICS REFERENCE SHEET

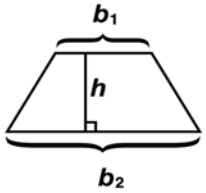
Pythagorean Formula

$c^2 = a^2 + b^2$



Trapezoid

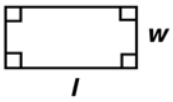
Area = $\frac{1}{2}h(b_1 + b_2)$



60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks = 1 year

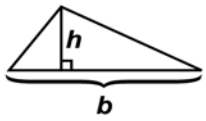
Rectangle

Area = lw
Perimeter = $2(l + w)$



Triangle

Area = $\frac{1}{2}bh$

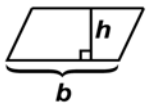


12 inches = 1 foot
3 feet = 1 yard
36 inches = 1 yard
5,280 feet = 1 mile
1,760 yards = 1 mile

100 centimeters = 1 meter
1000 meters = 1 kilometer

Parallelogram

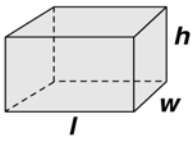
Area = bh



Circle

Area = πr^2
Circumference = $2\pi r$

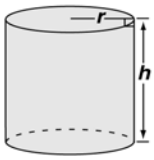
Rectangular Prism



Volume = lwh
Surface Area = $2lw + 2wh + 2lh$

Cylinder

Volume = $\pi r^2 h$
Surface Area = $2\pi rh + 2\pi r^2$

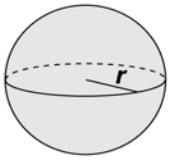


8 fluid ounces = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

1000 milliliters (mL) = 1 liter (L)

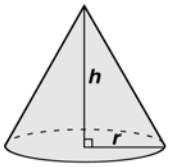
Sphere

Volume = $\frac{4}{3}\pi r^3$
Surface Area = $4\pi r^2$



Cone

Volume = $\frac{1}{3}\pi r^2 h$



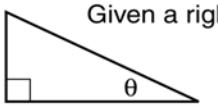
16 ounces = 1 pound

1000 milligrams = 1 gram
100 centigrams = 1 gram
10 grams = 1 dekagram
1000 grams = 1 kilogram

The sum of the measures of the interior angles of a triangle = 180°
The measure of a circle is 360° or 2π radians

$\pi \approx 3.14$ **or** $\frac{22}{7}$

Given a right triangle:



$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Interest = principal × rate × time

Simple Interest Formula: $A = p + prt$ **Compound Interest Formula:** $A = p\left(1+\frac{r}{n}\right)^{nt}$

A = amount after t years; p = principal; r = annual interest rate; t = number of years; n = number of times compounded per year

The number of **combinations** of n elements taken r at a time is given by $\frac{n!}{(n-r)!r!}$

The number of **permutations** of n elements taken r at a time is given by $\frac{n!}{(n-r)!}$

Given the points (x_1, y_1), (x_2, y_2),

Distance between two points:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

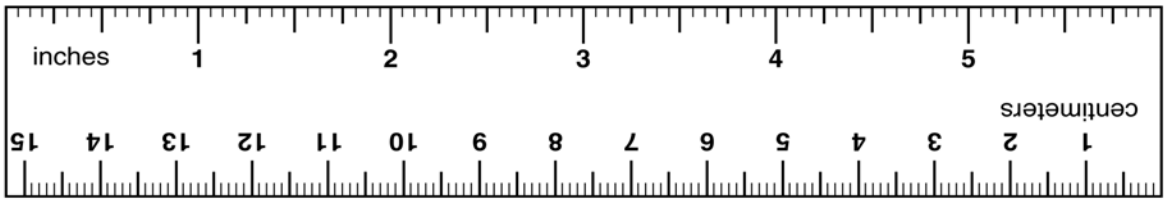
Slope Formula:

$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept form of a line:

$y = mx + b$

Distance = rate × time



STANDARD 1 – NUMBER AND NUMERICAL OPERATIONS

The following are two rational numbers greater than 1 and less than 2.

$$\frac{7}{6},\ 1.\overline{234}$$

- Give two more rational numbers greater than 1 and less than 2. Give reasons why your numbers are rational numbers.

The following are two irrational numbers greater than 1 and less than 2.

$$\frac{\pi}{2},\ 1.01001000100001...$$

- Give two more irrational numbers greater than 1 and less than 2. Give reasons why your numbers are irrational numbers.

Sample Solution:

- $\frac{5}{3}, \frac{4}{3}$ These numbers are rational numbers because they can be written as the ratio of two integers.
  OR
  1.5, 1.7689 These numbers are rational numbers as they have terminating decimals.
  OR
  $1.\overline{3}$, $1.\overline{51}$ These numbers are rational as they have repeating decimals.
  OR
  any ratio of integers, within the range of values, with correct reasoning
  OR
  any numbers, within the range of values, that have terminating decimals with correct reasoning
  OR
  any numbers, within the range of values, that have repeating decimals with correct reasoning

- 1.909009000900009…, 1.2468101214… These numbers are irrational as the decimal patterns do not repeat themselves.
  OR
  $\pi - 2$, $\frac{4\pi}{7}$ These numbers are irrational because π is an irrational number, and the sum/product of an irrational number and a rational number is irrational.
  OR
  $\sqrt{2}$, $\sqrt{3}$ These numbers have non-terminating, non-repeating decimals.
  OR
  any numbers, within the range of values, that have non-repeating non-terminating decimals with correct reasoning
  OR
  any sum or product of a rational number and an irrational number, within the range of values, with correct reasoning

Scoring Rubric

3-Point Response
The response contains:

- two correct rational numbers and correct reasoning why the numbers are rational
  AND
- two correct irrational numbers and correct reasoning why the numbers are irrational.

2-Point Response
The response contains:

- one correct rational number with correct reasoning
  AND
- one correct irrational number with correct reasoning
  OR
- two rational numbers, which do no necessarily fall into the range of values, with some reasoning
  AND
- two irrational numbers, which do not necessarily fall into the range of values, with some reasoning
  OR
- four correct values with reasoning for at least one of them.

1-Point Response
The response contains:

- four correct values with no correct reasoning
  OR
- one correct value with reasoning
  OR
- two values that are correctly labeled as rational or irrational but do not necessarily fall within the range, and at least one of these values contains reasoning.

0-Point Response

- The response demonstrates insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the required solutions, or the explanation may not be understandable. How decisions were made may not be readily understandable.

MATHEMATICS
OPEN-ENDED RESPONSE

Two rational num >1 but <2 are

1.5 and $1.\overline{789}$,

→ These two numbers are rational because they have definate patterns and do not continue on forever. They can also be converted into fractions.

Two irrational numbers >1 but <2 are

$\frac{\pi}{3}$ and 1.0219518...

→ These two numbers are irrational because they continue on forever with no definate pattern, ∴ they cannot be converted into fractions.

Score Point: 3
The response contains two correct rational numbers with explanation—"can also be converted into fractions." The response contains two correct irrational numbers with explanation—"continue on forever with no definite pattern, they cannot be converted into fractions."

MATHEMATICS
OPEN-ENDED RESPONSE

- $\frac{4}{3}$, $1.\overline{689}$ These numbers are rational because they do not produce a whole number, (A) and have a repeating pattern. To further explain $\frac{4}{3}$, when simplified, is $1.33333\overline{3}$. This number will neither end, nor discourage from this pattern. $1.\overline{689}$ is another example because its pattern will never end, but it won't stray from the pattern: $1.689689\overline{689}$.

Non-terminating
Repeating

- $\frac{\pi}{3}$, 1.0200200002... These numbers are irrational because they do not produce a whole number, are non-terminating, and non-repeating. To further explain $\frac{\pi}{3}$, when simplified, yields 1.047197551... This number never terminates and does not repeat any pattern, whatsoever. 1.0200200002... neither repeats a sequence nor terminates ever.

Non-terminating
Non-repeating

(A) non-terminating

Score Point: 3
The response contains two correct rational numbers with explanation—"have a repeating pattern." The response contains two correct irrational numbers with explanation—"non-terminating and non-repeating."

MATHEMATICS
OPEN-ENDED RESPONSE

- $1.\overline{66}$ and $\frac{5}{4}$

they are rational because $\frac{5}{4}$ terminates and $1.\overline{66}$ has a repeating decimal

- $\frac{\pi}{3}$ and 1.0200020000 2...

$\frac{\pi}{3}$ infinately continues ($\pi = 3.14...$) and 1.0200020000 2... does not have a repeat decimal. Neither terminates so they are both irrational numbers.

Score Point: 3

The response contains two correct rational numbers with explanation—"5/4 terminates and $1.\overline{66}$ has a repeating decimal." The response contains two correct irrational numbers with explanation—"does not have a repeat decimal. Neither terminates."

MATHEMATICS
OPEN-ENDED RESPONSE

- 1.5 it has an end point
  1.25 it has an end point
- $\sqrt{2}$ The number has no end
  $\sqrt{3}$ The number has no end.

Score Point: 2
The response contains two correct rational numbers with explanation—"it has an endpoint"—interpreted as the number terminates. The response contains two correct irrational numbers; however, the explanation is flawed—"the number has no end"—but does not mention it does not repeat.

MATHEMATICS
OPEN-ENDED RESPONSE

$5/3$ is a rational number less than two and greater than one because it is a number with a repeating decimal making it a rational number, and because it is between the numbers one and two.
1.5 is a rational number less than two and greater than one because it has a terminating decimal which makes it rational, and because it is in between the numbers one and two.

$\frac{457}{456}$ is greater than one and less than two, and it is rational because the decimal continues forever and doesn't repeat.

1.047197551... is greater than one and less than two, and it is irrational because the decimal keeps continuing and doesn't repeat.

Score Point: 2
The response contains two correct rational numbers with appropriate explanations for each—"repeating decimal," "terminating decimal." The response contains two correct irrational numbers with explanation; however, the student identifies one of the numbers as rational.

MATHEMATICS
OPEN-ENDED RESPONSE

① $\frac{3}{2}$, $\frac{7}{5}$. These numbers are rational because they terminate in decimal form.

② $\frac{4}{\pi}$, 1.8754983... These numbers are irrational because now matter how far you extend the numbers after the decimal point, they will never terminate and never be definite.

Score Point: 2
The response contains two correct rational numbers with appropriate explanation—"they terminate in decimal form." The response contains two irrational numbers; however, the explanation is flawed as it does not mention that each of the numbers does not have a repeating decimal.

MATHEMATICS
OPEN-ENDED RESPONSE

- $\frac{6}{5}$, 1.5

- $\frac{\pi}{3}$, 1.45678101112 13 - - -

Score Point: 1
The response contains two correct rational numbers and two correct irrational numbers. However, an explanation for each set is missing.

MATHEMATICS
OPEN-ENDED RESPONSE

- 2 rational numbers

  1.834 and $\frac{9}{8}$

  the reason these numbers are rational is because they are unrepetitive and have a stop.

- 2 irrational numbers

  1.08080808... & $\frac{8}{9}$

  These numbers are irrational because they repeat and keep going.

Score Point: 1

The response contains two correct rational numbers with explanation—"have a stop." The response contains two numbers that are labeled irrational; however, they are both rational as each of them can be expressed as a fraction (repeating pattern). Also, one of the numbers (8/9) does not fall into the correct range of values.

MATHEMATICS
OPEN-ENDED RESPONSE

Two rational numbers greater than 1 and less than 2

$\frac{8}{5}$ and $1.\overline{12}$

The first number is rational because it is an easy fraction to deal with and divides out to a real number. The second number is rational because it is also a real number and has a constant pattern.

Two irrational numbers greater than 1 and less than 2:

$\frac{\pi}{1.5}$ and $\frac{\pi}{3}$

Both of these numbers are irrational because any number that divides π will be irrational. π cannot be defined easily.

Score Point: 1

The response contains two correct rational numbers; however, it only explains one of them—"a real number and has a constant pattern." The response contains two irrational numbers; however, one of the numbers is outside the range of values. The explanation for irrational numbers is incorrect.

MATHEMATICS
OPEN-ENDED RESPONSE

- $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2} = 1.5$

  m $\frac{3}{2} = 1\frac{1}{2}, 1.5$

  $\frac{5}{3}, 1.6$

  $\frac{3}{2}$ is greater than 1 and less than 2

  $\frac{5}{3}$ is greater than 1 and less than 2

  reason Whole #'s

- $\frac{3.17}{2}, 1.428571429$

  no continuing pattern
  fraction is decimal

Score Point: 0
The response contains two correct rational numbers with an incorrect explanation—"whole #'s." The response contains another two values which are not irrational.

MATHEMATICS
OPEN-ENDED RESPONSE

Use this page for question 48 only.

48. $\frac{9}{8}$ and $1.\overline{345}$ are rational. They are rational because they can be reduced and/or divided.

$\frac{\pi}{1}$ and $\sqrt{\frac{2}{7}}$ are irrational. They are irrational because they cannot be reduced and/or divided.

Score Point: 0

The response contains two correct rational numbers with an incorrect explanation—"they can be reduced and/or divided." The response contains two irrational numbers that do not fall into the range of values and do not have a correct explanation—"cannot be reduced and/or divided."

MATHEMATICS
OPEN-ENDED RESPONSE

>1 <2

$\frac{7}{6}$, 1.234

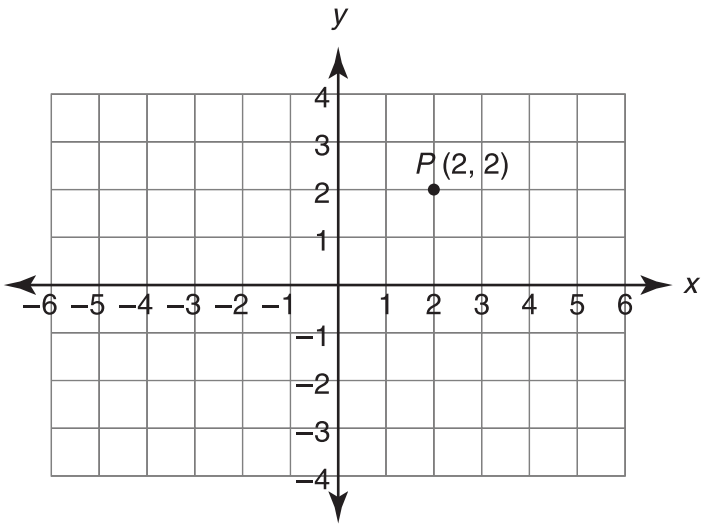
- TWO more rational

$1.\overline{345}$ $\frac{8}{6}$ = more then 1 - real numbers
less then 2 less then 2
more then 1

$1.\overline{44444}$ - go on ~~forever~~ (never stop)
1.344 - never stop.

Score Point: 0
The response contains two correct rational numbers with no explanation. The response does not contain two correct irrational numbers.

STANDARD 2 – GEOMETRY AND MEASUREMENT



Raul's teacher told him that the order in which two transformations are performed could affect the final image. If Raul reflected point P (2, 2) over the y-axis and then translated the image two units to the right, the final image of P would be P'' (0, 2).

- Would the final image have been any different if the point had first been translated two units to the right and then reflected over the y-axis? Support your answer by giving the coordinates of the final image.

Raul's teacher wrote the following three transformations on the board:

Reflect over the x-axis
Reflect over the y-axis
Translate 2 units up

Use the transformations in the list above and the point in the diagram to answer the following.

- Choose two of the transformations from the list above for which the final image would be affected by the order in which the transformations are performed. Support your answer by giving the coordinates of both of the final images, one for each order in which the transformations are performed.

- Choose two of the transformations from the list above for which the final image would not be affected by the order in which the transformations are performed. Support your answer by giving the coordinates of the final image of both of the orders in which the transformations are performed.

Sample Solution:

- Yes, the image would be affected. The coordinates of the image of the first set of transformations would be (0, 2). The coordinates of the image of the second set of transformations would be (−4, 2).

- Reflect over the *x*-axis and translate two units up. The image of reflecting and then translating would be the point (2, 0). The image of translating and then reflecting would be (2, −4).

- Reflect over the *x*-axis and then reflect over the *y*-axis. The final image of both orders of reflection would be the point (–2, –2).

Scoring Rubric

3-Point Response
The response contains:

- the correct answer of yes, the image would be affected by the order, and the image of the point
  AND
- the correct pair of transformations for which order affects the final image and the image of the point, using both orders
  AND
- a correct pair of transformations for which the images would not be affected by the order in which the transformations were performed
  AND
- the final image of this pair of transformations.

2-Point Response
The response contains:

- the correct answer of yes, the image would be affected by the order, and the image of the point
  AND
- the correct pair of transformations for which order affects the final image
  AND
- a correct pair of transformations that would not be affected by the order, but the images of the transformations are missing
  OR
- two correct answers with the correct images.

1-Point Response
The response contains:

- one correct answer with the correct images
  OR
- a minimal understanding of the concepts—the response contains errors in finding the images that lead to an incorrect answer.

0-Point Response

- The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution, or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

MATHEMATICS
OPEN-ENDED RESPONSE

Use this page for question 48 only.

48.

Yes, if you translate Point P 2 units to the right befor you reflect it over the y axis, you end up with (-4, 2) not (0, 2).

2 transformations that would effet the final answer due to thier order would be reflect over the X-axis and Translate 2 units up.

If you translate first, your result will be (2, -4) but if you reflect first, your answer will be (2, 6)

2 transformations that won't affect the answer by the order the are in are Reflet over the x-axis and reflect over the y axis. either way, you still end up with (-2, -2)

Score Point: 3

The response answers all bullets correctly with the correct images.

MATHEMATICS
OPEN-ENDED RESPONSE

- yes the final image would be different.
the coordinates would then be (-4, 2).

- reflect over x-axis then translate 2 units up
final image would be (2, 0)
if order switched the final image would be (2, -4)

- reflect over y-axis then translate up 2 units
final image would be (-2, 4)
translate up 2 units then reflect over y-axis
final image would be (-2, 4)

Score Point: 3
The response answers all bullets correctly with the correct images.

MATHEMATICS
OPEN-ENDED RESPONSE

- Yes the final image would have been different if Raul had translated the point first If he had translated first and then reflected the final image would have been P'(4,2) P''(-4,2).
- Reflection over the x axis followed by translation of 2 units up would produce a final image of P''(2,0) If P had been translated first, the final image would have been P''(2,-4) This is an example in which the order of the transformations would affect the final image.
- A Reflection over the x-axis followed by a reflection over the y-axis would produce a final image P''(-2,-2) If reflected across the y axis and then the x-axis the final image P''(-2,-2), This example shows that the final image is not affected by the order of the transformations.

Score Point: 3
The response answers all bullets correctly with the correct images.

MATHEMATICS
OPEN-ENDED RESPONSE

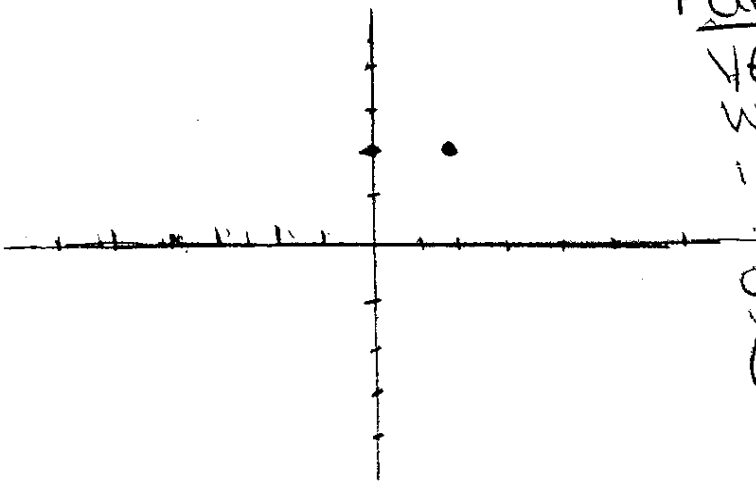
Yes they would be different. If Ray would had move to the right first then flip it over the Y axes. With move down to the right first then flipped of they Yaxes the coordinates is now (-4, 2).

The image starts at plot (0, -1). Then I will flip the point over the Y axes to move to (0, 1). After that I will translate the plot to units up to give it's final spot (0, 3). Now if instead of flipping over the axes first I will translate 2 units up it will move to point (0, 1) to point (0, 1) then flip over the axes it will stop a (0, -1).

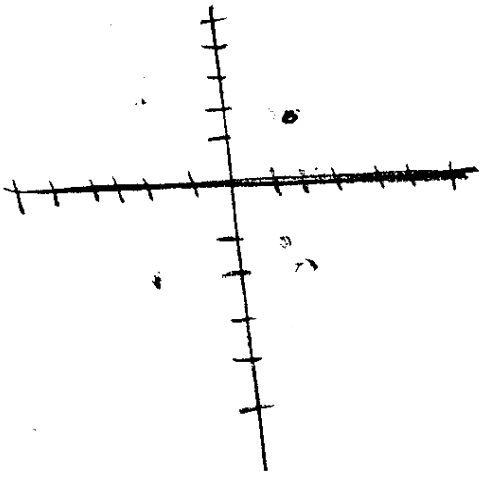
Score Point: 2

The response contains the correct answer to the first bullet with the correct image. The student correctly answers the second bullet. Even though a different point was chosen, the correct transformation and final images are given. The third bullet was not attempted.

MATHEMATICS
OPEN-ENDED RESPONSE



Part 1: (-4,2)
Yes because when you moved it to the right twice it would go to corrclintes (4,2) then you flip it over the y-axis and it would be (-4,2)



Part 2: Reflect over the x-axis & Translate 2 units up if you do it in that order you come w/ (0,2) if you switch then you get (-4,2).

Part 3: if you did reflect over the y-axis and then reflect over the x-axis you get (-2,2) even if you switch them you get (-2,2)

Score Point: 2

The response contains the correct answers to the first and third bullets, with the correct images of the point. In bullet two, the response contains a correct pair of transformations; however, the image of each point after the transformation is incorrect.

MATHEMATICS
OPEN-ENDED RESPONSE

- The final image would have beed different
  The final points are (-4, 2)

- Reflect over the y axis, reflect over the x axis
  (-2, -2)

- reflect over the x axis, reflect over the y axis
  (-2, -2)

Score Point: 2
The response contains the correct answers to the first and third bullets, with the correct images of the point.

MATHEMATICS
OPEN-ENDED RESPONSE

• yes, the final image would have been affected. If P(2, 2) was first translated two units to the right, it would become (4, 2). When it is reflected the final image of P would be P"(-4, 2).

The final image of the transformations above would be (-2, 0).

• If you were to translate 2 units up first, then reflect over the y axis, and then the x, you would end up with (-2, -4) rather than (-2, 0)

If you translate 2 units up, reflect over the x-axis, and then over the y-axis, the final image would be (-2, -4) also.

• If you were to reflect over the y axis instead of the x first then over the x-axis and finally 2 units up, the coordinates of the point would be (-2, 0). The final image was not affected.

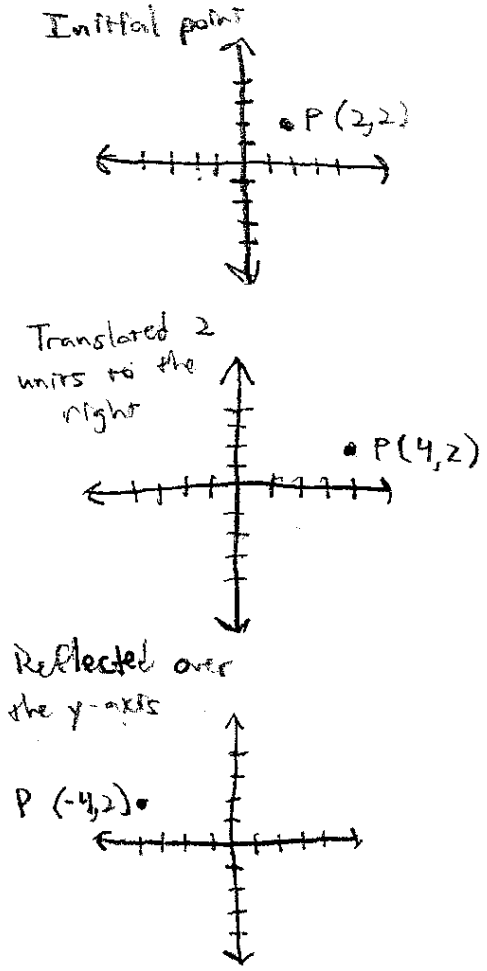
-If you reflected over the x-axis, moved 4 units up, then over the y, your final image would be (-2, 0). This would be unaffected

Score Point: 1

The student correctly answers the first bullet with the correct image of the point. The responses to the second and third bullets contain three transformations.

MATHEMATICS
OPEN-ENDED RESPONSE

- Yes the coordinates would have been (-4, 2)



- Translate P up 2 units. Then reflect over y-axis. The coordinates are (2, 4). If done in reverse order the coordinates are (2, -4)

- Translate P up 2 units. Then reflect over x-axis. The coordinates are (-2, 4). If done in reverse order the coordinates are still (-2, 4)

Score Point: 1
The student correctly answers the first bullet with the correct image of the point. The response to the second bullet is incorrect. The response to the third bullet is also incorrect.

MATHEMATICS
OPEN-ENDED RESPONSE

The final image would be different if it was translated first.
P(2,2) translated would be (4,2)
Reflected – P"(-4,2)

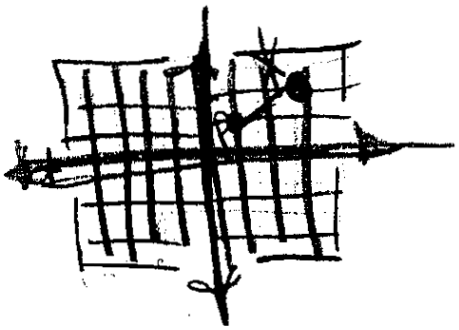
- Affected: Reflects over x-axis (2,-2) Then 2 units to the right P"(4,-2)
  Translate 2 units up (2,4) Reflected P"(-2,4)
- Not affected: Reflects over y-axis the two units right (-2,2) P"(0,2)

Score Point: 1
The student correctly answers the first bullet with the correct image of the point. The response to the second bullet is incorrect. The response to the third bullet is also incorrect.

MATHEMATICS
OPEN-ENDED RESPONSE

Use this page for question 48 only.

48. Yes, the final image would have been different. The pts would have been in another area. It would have been longer



Score Point: 0

The response contains the correct answer to only the first bullet; however, the image of the point is not provided.

MATHEMATICS
OPEN-ENDED RESPONSE

- P(2,2) y axis trans. 2 to right
  (0,2)
  yes (4,2) would have been the Final image

- Reflect over y axis (2,2) Final
  Reflect over x axis (4,2) Final

- translating the point 2 units up would not
  affect it because the transition is already complete

Score Point: 0
The response contains the correct answer to only the first bullet; however, the correct image of the point is not provided.

MATHEMATICS
OPEN-ENDED RESPONSE

- Yes, the final image of P(2,2) would have been different if it were translated two units to the right and reflected over the y-axis because P" would land on (2, -4).

- From the list, the final image P"(0,2) would be affected if it was translated 2 units up and reflected over the x-axis, giving the final image of P, (0,-4).

- From the list, if the final image P"(0,2) was reflect over the y-axis and moved 2 units up it would remain on (0,2).

Score Point: 0
The response contains the correct answer to only the first bullet; however, the correct image of the point is not provided.

STANDARD 3 – PATTERNS AND ALGEBRA

For each bicycle that it repairs, a repair shop charges for parts and $35 per hour for labor.

- Write an equation for the total charge, *C*, of a repair with the cost of parts, *p*, and the number of hours of labor, *n*.

- The shop adds a 6% tax on the total charge for each repair. Write an equation for the total charge, *T*, after tax, of a repair with the cost of parts, *p*, and the number of hours of labor, *n*.

- The total charge after tax of a bicycle repair was $233.20. The cost of the parts was $80. How many hours of labor were charged in this bicycle repair? Show your work or provide an explanation for your answer.

Sample Solution:

- $C = 35n + p$

- $T = (35n + p)1.06$
  OR
  $T = 35n + p + (35n + p)0.06$
  OR
  $T = 37.10n + 1.06p$

- 4 hours
  $233.20 = (35n + 80)1.06$
  $220 = 35n + 80$
  $140 = 35n$
  $4 = n$

  OR

  $233.20 = 37.10n + 84.80$
  $148.40 = 37.10n$
  $4 = n$

  OR

  The total price for 1 hour of work is $121.90.
  The total price for 2 hours of work is $159.00.
  The total price for 3 hours of work is $196.10.
  The total price for 4 hours of work is $233.20.

Scoring Rubric

3-Point Response
The response contains:

- the correct equation for the cost before tax
  AND
- the correct equation for the total cost after tax
  AND
- the correct number of hours, with work or explanation to support the answer.

2-Point Response
The response contains:

- two correct equations
  AND
- an incorrect answer for the third part, or no work for the third part
  OR
- a correct first equation
  AND
- an error in the second equation
  AND
- an answer to the third part that is correct based on the given incorrect second equation with work/explanation, OR the correct answer to the third part with correct work or explanation for this answer
  OR
- an error in the first equation (the response has the first equation multiplied by 1.06 as the second equation).
  AND
- the given second equation is used correctly to find the answer to the third part with correct work/explanation, OR the response contains the correct answer to the third part with correct work or explanation for this answer.

1-Point Response
The response contains:

- the correct first equation
  OR
- the first equation multiplied by 1.06 as the answer to the second equation
  OR
- the student correctly uses the second equation to find an answer with work or explanation for how the answer was found
  OR
- the correct answer to the third part with work or explanation for how this answer was found.

0-Point Response

- The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the required solutions, or the explanation may not be understandable. How decisions were made may not be readily understandable.

MATHEMATICS
OPEN-ENDED RESPONSE

A) $C = p + 35n$

B) $T = 1.06(p + 35n)$

C) $T = 233.20 =$ total cost

$p = 80 =$ parts money

find n

$233.20 = 1.06(80 + 35n)$

$233.20 = 84.8 + 37.1n$

$-84.8 \quad -84.8$

$\frac{148.4}{37.1} = \frac{37.1n}{37.1}$

$4 = n$

The total amount of hours worked is 4

Score Point: 3

The response contains the correct equation for the cost before tax, the correct equation for the total cost after tax, and the correct number of hours, with work or explanation to support the answer.

MATHEMATICS
OPEN-ENDED RESPONSE

$- C = p + 35n$

$- T = (p + 35n)1.06$

$- 233.20$

1.06 ← removes tax

220

$- 80$ ← removes parts cost

140

35 ← cost of labor

4 hours of labor.

Score Point: 3
The response contains the correct equation for the cost before tax, the correct equation for the total cost after tax, and the correct number of hours, with work or explanation to support the answer.

MATHEMATICS
OPEN-ENDED RESPONSE

- $C = P + 35n$
- $T = 1.06(P + 35n)$

$1.06\overline{)233.20}$ = 220

$220 - 80 = 140$

$35\overline{)140}$ = 4

There was 4 hours of labor spent on this bike. First you must get rid of the tax for the repair by dividing 233.20 by 1.06. Next you take that answer & subtract 80 from it. That will give you how much the total hours cost which is $140. Last you take the 140 & divide it by the hourly cost of $35 to give you the # of hours spent on the bike.

Score Point: 3
The response contains the correct equation for the cost before tax, the correct equation for the total cost after tax, and the correct number of hours, with work or explanation to support the answer.

MATHEMATICS
OPEN-ENDED RESPONSE

$C = p + 35n$

$T = .06(p + 35n) + (p + 35n)$

$\$233.20 = .06(\$80 + \$35n) + (\$80 + \$35n)$

$\$3886.67 = (80 + 35n) + (80 + 35n)$

$-35n - 35n = 160 - 3886.67$

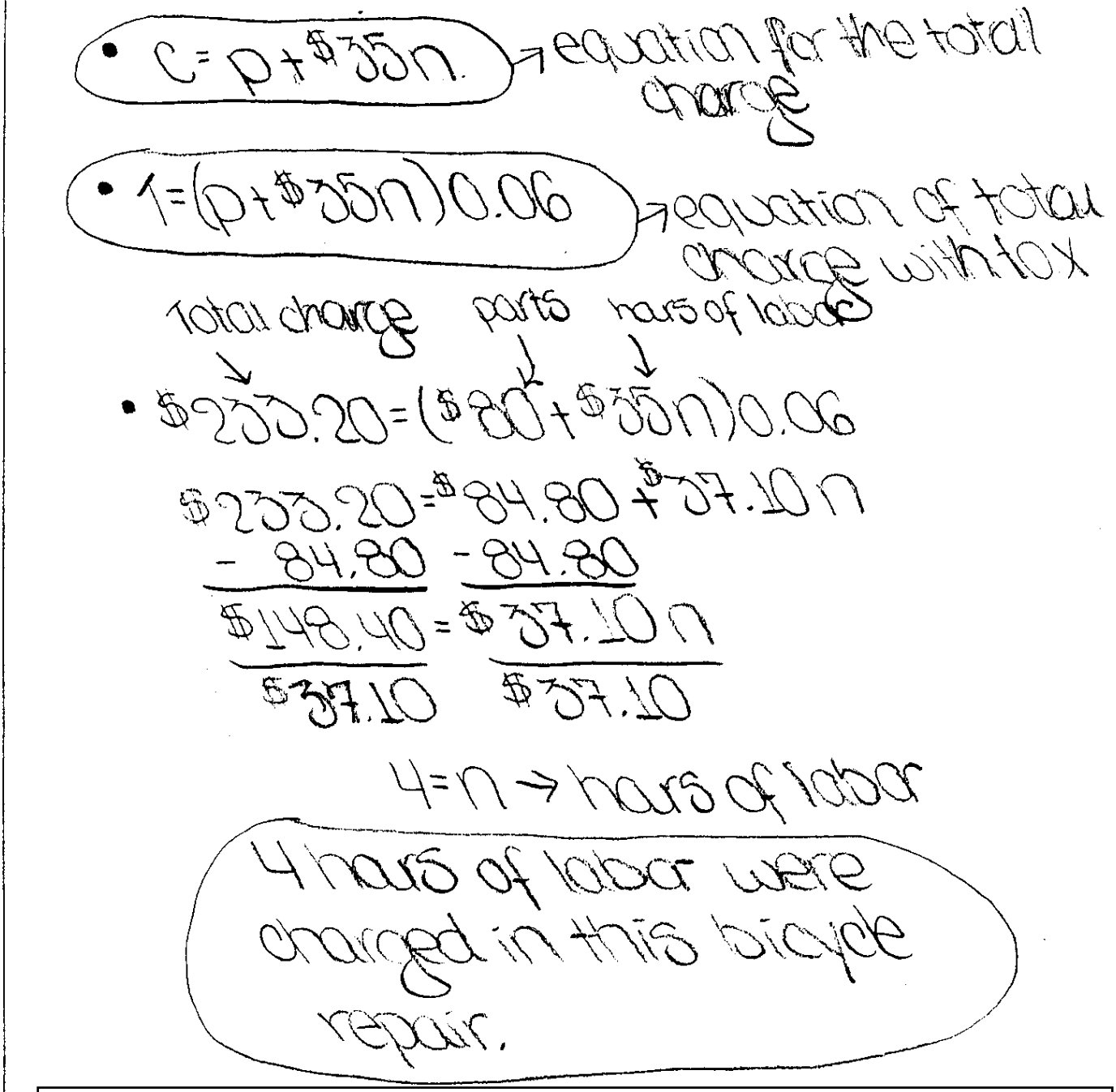
~~$n = 53.2$~~

Used trial and error 4 hours

Score Point: 2

The response contains the correct equation for the cost before tax and the correct equation for the total cost after tax. Even though the student states the correct number of hours, the work is flawed. An explanation of "trial and error" is not enough to support the answer, unless the "trial and error" work is shown.

MATHEMATICS
OPEN-ENDED RESPONSE



Score Point: 2

The response contains the correct equation for the cost before tax. The response for the second equation is flawed—the student multiplied by 0.06 instead of 1.06. However, the response for the third bullet is correct because the student multiplied by 1.06, even though he/she wrote 0.06, and obtained the correct answer.

MATHEMATICS
OPEN-ENDED RESPONSE

Part 1

$C = 35n + P$

Part 2

$T = C + (C \cdot .06)$

Part 3

$\frac{233.20}{.06} = (35n + 80) + (35n + 80 \cdot .06)$

$\frac{70n + 160 \cdot .06}{.06}$

$3883.33 = 70n + 160$

$-160 \quad -160$

$\frac{3723.33}{70} = \frac{70n}{70}$

$53.19 = n$

Approximately 53.19 hours of labor were charged

Score Point: 2

The response contains the correct equation for the cost before tax. The response contains the correct equation for the total cost after tax. The student identifies the variable *C* in the first equation and uses it appropriately for the second equation. The response for the third bullet is flawed.

MATHEMATICS
OPEN-ENDED RESPONSE

- $C = P + 35n$
- $T = P + 35n + (P + 35n \times \frac{6}{100})$
- $233.20 = T$

  $80 = P$

$233.20 = 80 + 35n + (80 + 35n \times .06)$

$233.20 = 80 + 35n + 80 + 35n \times .06$

$233.20 \times .06 = 80 + 80 + 35n + 35n$

$113.992 = 160 + 70n$

$173.992 = 70n$

$n = 2.48$

The work was done for 2 hour 48 min

Score Point: 1

The response contains the correct equation for the cost before tax. The response for the second bullet is flawed. The response for the third bullet is also flawed.

MATHEMATICS
OPEN-ENDED RESPONSE

- $C = p + 35n$
- $C = p + 35n + t$
- $233.20 - 13.99 = 219.21$

  $219.21 - 80.00 = 139.21$

  $35 \times 4 = 140$

  4 hour of hard labor

Score Point: 1

The response contains the correct equation for the cost before tax. The response for the second bullet is incorrect. The response contains the correct answer for the third bullet; however, the work is flawed.

MATHEMATICS
OPEN-ENDED RESPONSE

- $C = p + 35n$
- $T = 6\%.(p+35n)$
- $T = \$233.20$
  $p = \$80$

$\$233.20 = 6\%.(80 + 35n)$

Score Point: 1
The response contains the correct equation for the cost before tax. The responses for the second and third bullets are incorrect.

MATHEMATICS
OPEN-ENDED RESPONSE

$C = pn$

c = total charge

p = parts

n = # charge/hour labor

n = 35/hour

$C = P(35)$

T = total charge w/ tax

$C(1.06) = P(n)$; $C(1.06) = P(35)$

$C(1.06) = T$

$T = P(n) = T = P(35)$

w/ tax

$233.20 = 80n$

w/o tax

$\frac{220.00}{80} = \frac{80n}{80}$

$2.75 \approx n$

2.75 numbers of labor

Score Point: 0

The response is incorrect for each of the three bullets.

MATHEMATICS
OPEN-ENDED RESPONSE

- $35 per hour for labor

- C + P + N=

  $233.20. C + $80. P + $35 N

- 233.70. $139.92 C + $48 P + 21 N = 23

  x60% → Charge

  139.92

  $80. → Parts $48 P

  x60%

  48

  $35 → Labor $21 N

  x60%

  21

- 80 ÷ 35 = 2.28 hours

  Divide the Price was $80 ÷ 35 = 2.28 hours of Labor

Score Point: 0
The response is incorrect for each of the three bullets.

MATHEMATICS
OPEN-ENDED RESPONSE

- n = #s of hours in labor
  p = repair with cost parts
  C = Total change

  $C = p(n)$

  $36 = p(n)$

- T = total change
  p = repair with cost parts
  n = # of hours in labor

  $36 \times 1.06 = 38.16$

  $T = p(n)$

  $38.16 = p(n)$

- Total change = $233.20
  Cost parts = 80
  hours of labor = x

  $233.20 = 80(x)$

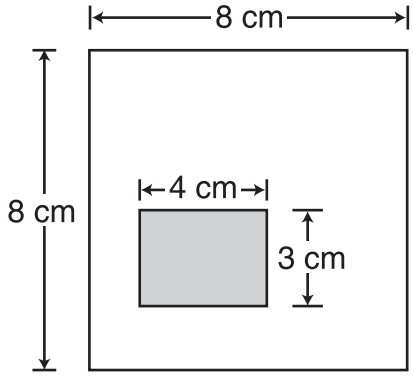
  $\frac{233.20}{80} = \frac{80x}{80}$

  $x = 2.92$

  $x = 3.52$ hrs

Score Point: 0
The response is incorrect for each of the three bullets.

STANDARD 4 - DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS



The 8-cm by 8-cm square region, shown in the diagram above, contains a shaded 4-cm by 3-cm rectangular region. A computer program is designed to randomly choose a point inside the 8-cm by 8-cm square region.

- The computer chooses 1,000 points inside the square. Approximately how many of the points would be expected to fall inside the shaded rectangle? Show your work or provide an explanation for your answer.

- A second 8-cm by 8-cm square region is created containing a different shaded rectangular region. The computer chose 1,000 points inside the square region. Of the 1,000 points chosen, 250 of the points were inside the shaded rectangular region. Approximate the area of the shaded rectangular region. Show your work or provide an explanation for your answer.

- Give one set of possible dimensions for the second shaded rectangular region.

Sample Solution:

- Accept whole number answers in the range 175 – 200.
  $\frac{3 \times 4}{8 \times 8} \times 1,000 = 187.5$

- 16 square cm (Accept answers in the range of 14 square cm to 18 square cm.)
  $\frac{250}{1,000} = \frac{x}{64}, x = 16$

- 4 cm by 4 cm OR 8 cm by 2 cm
  $4 \times 4 = 16$ OR $8 \times 2 = 16$

Scoring Rubric

3-Point Response
The response contains:

- a reasonable estimate of the number of points that will fall within the rectangle with clear, correct work or explanation for the answer
  AND
- a reasonable estimate of the area within the rectangular region with clear, correct work or explanation for the answer
  AND
- correct dimensions for the area that was calculated with neither dimension greater than 8 cm.

2-Point Response
The response contains:

- three correct answers with work or explanation for at least one of them
  OR
- two correct answers with work or explanation for both of them; the third part will be considered to be correct if the dimensions given multiply together to equal the area calculated and are less than or equal to 8 cm.

1-Point Response
The response contains:

- two correct answers
  OR
- a reasonable number of points with work or support for the answer
  OR
- one correct answer with work or explanation; the third part will be considered correct if the dimensions given multiply together to equal the area calculated and are less than or equal to 8 cm
  OR
- dimensions which, when multiplied together, give the area calculated, and each of which is less than or equal to 8 cm.

0-Point Response

- The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution, or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

MATHEMATICS
OPEN-ENDED RESPONSE

$\frac{12\text{cm}^2}{64\text{cm}^2} = 3/16$

$3/16 \cdot 1000 = 187.5$

188 times

$\frac{x}{64} = \frac{250}{1000}$ $250 \cdot 64 = 1000x$

$x = 16\text{cm}^2$

C. 8cm by 2-cm

Score Point: 3
The response contains a correct answer to each of the three bullets, with work or explanation as required.

MATHEMATICS
OPEN-ENDED RESPONSE

- $1000 \cdot .1875 = 187.5$ 188 points

If the computer choose 1000 points approximately 188 points will land in the shaded area because the shaded area is 18% of the square and 188 is 18% of 1000.

16 cm^2

- The area of the shaded region would be 16 because 250 is 25% of 1000 and 16 is 25% of 64.

- One set of possible dimensions is 4cm by 4cm.

Score Point: 3
The response contains a correct answer to each of the three bullets, with work or explanation as required.

MATHEMATICS
OPEN-ENDED RESPONSE

$8 \times 8 = 64$ $\quad \frac{12}{64} = .1875 = 18.75\%$ $\quad 1000 \cdot .1875 \approx 188$

$4 \times 3 = 12$

I believe about 188 dots will fall into the shaded region.

I got this by getting the area of the figures, I got the percentage of the smaller as compared to the larger one, and got 18.75%. Then I simply found out what 18.75% of 1000 was.

The area is about $16 in^2$. I got this by establishing that 250 is $\frac{1}{4}$ of 1000. Using this knowledge I took $\frac{1}{4}$ the area of the 8×8 square. Thus I got my answer of 16.

4×4 is a possible dimension, seeing that it equals sixteen, $\frac{1}{4}$ of 64.

Score Point: 3
The response contains a correct answer to each of the three bullets, with work or explanation as required.

MATHEMATICS
OPEN-ENDED RESPONSE

- About 188 points will land in the shaded rectangle out of the 1,000 points chosen. This is because regularly the probability that a point would be chosen from the shaded region would be 3 out of 16. 16 goes into 1,000 62.5 times; $1000 \div 16 = 62.5$. $3 \times 62.5 = 187.5$. So it is about 188 times.
- The area of the shaded rectangular region is 16. If the sides are 1 by 16, the probability is 16 out of 64 or 1 out of 4. $1000 \div 4 = 250$ $250 \times 1 = 250$.
- One set of possible dimensions for the second shaded rectangular region is 1 by 16.

Score Point: 2
The response contains a correct answer to each of the first two bullets, with work or explanation. The response to the third bullet is incorrect as one of the dimensions is greater than 8.

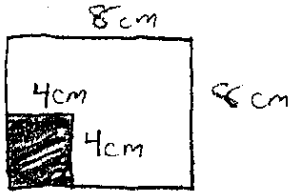
MATHEMATICS
OPEN-ENDED RESPONSE

• Approximately 273 points would fall inside the shaded rectangle.

12cm:64cm
3:11

$1000 \cdot (3/11) = 273$

• The approximate area of the shaded rectangular reigon would be about 16 cm



250 is 1/4 of 1000, so the shaded rectangular reigon would be 1/4 the area of the bigger square reigon.

16cm:64cm 1:4

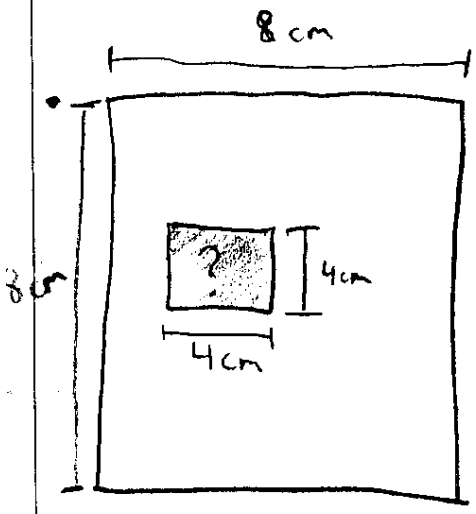
• The possible demensions for the second shaded reigon could be 4 cm by 4cm.

Score Point: 2
The response contains an incorrect answer to the first bullet. The response contains correct answers to the second and third bullets, with work or explanation as required.

MATHEMATICS
OPEN-ENDED RESPONSE

- $8cm \times 8cm = 64cm$ = Total Area.
  $4cm \times 3cm = 12\ cm$ = Shaded Area.

  $64 \div 12 = 5.3$ = Probability of a point landing in shaded area.
  1000 points $\div 5.3 =$ (188.7) = Number of points that will land in shaded area.



1000 points $\div 250$ points $= 4 = \frac{1}{4}$ probability of a point landing in the shaded region

$64cm \div \frac{1}{4} = 16cm$ = area of shaded region

$16\ cm = 4cm \times 4cm$ = Shaded Area

$64cm = 8\ cm \times 8cm$ = Total Area

- The 2nd shaded region is $4cm \times 4cm$

Score Point: 2
The response contains a flaw in the first bullet. The work is correct; however, the student does not round the answer to a whole number. The response contains correct answers to the second and third bullets, with work or explanation as required.

MATHEMATICS
OPEN-ENDED RESPONSE

$\frac{1000}{64} = \frac{x}{52}$

$\frac{52000 = 64x}{64 \quad 64}$

$x = 812.5$

1000
- 812.5
187.5

I would expect 187.5 points inside of the shaded box, I found this by the ratio $\frac{1000}{64} = \frac{x}{52}$ then cross multiplying and solving for x. Then I took the 812.5 I got and subtracted that number from 1000 giving me the possibility of 187.5 points.

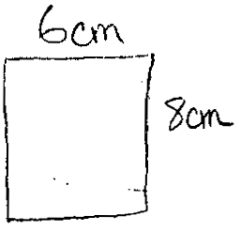
$\frac{1000}{64} = \frac{x}{48}$

$48000 = 64(750)$

1000
- 750
250

I believe the area of the shaded area is 48cm I found this by going backwards from the equation above

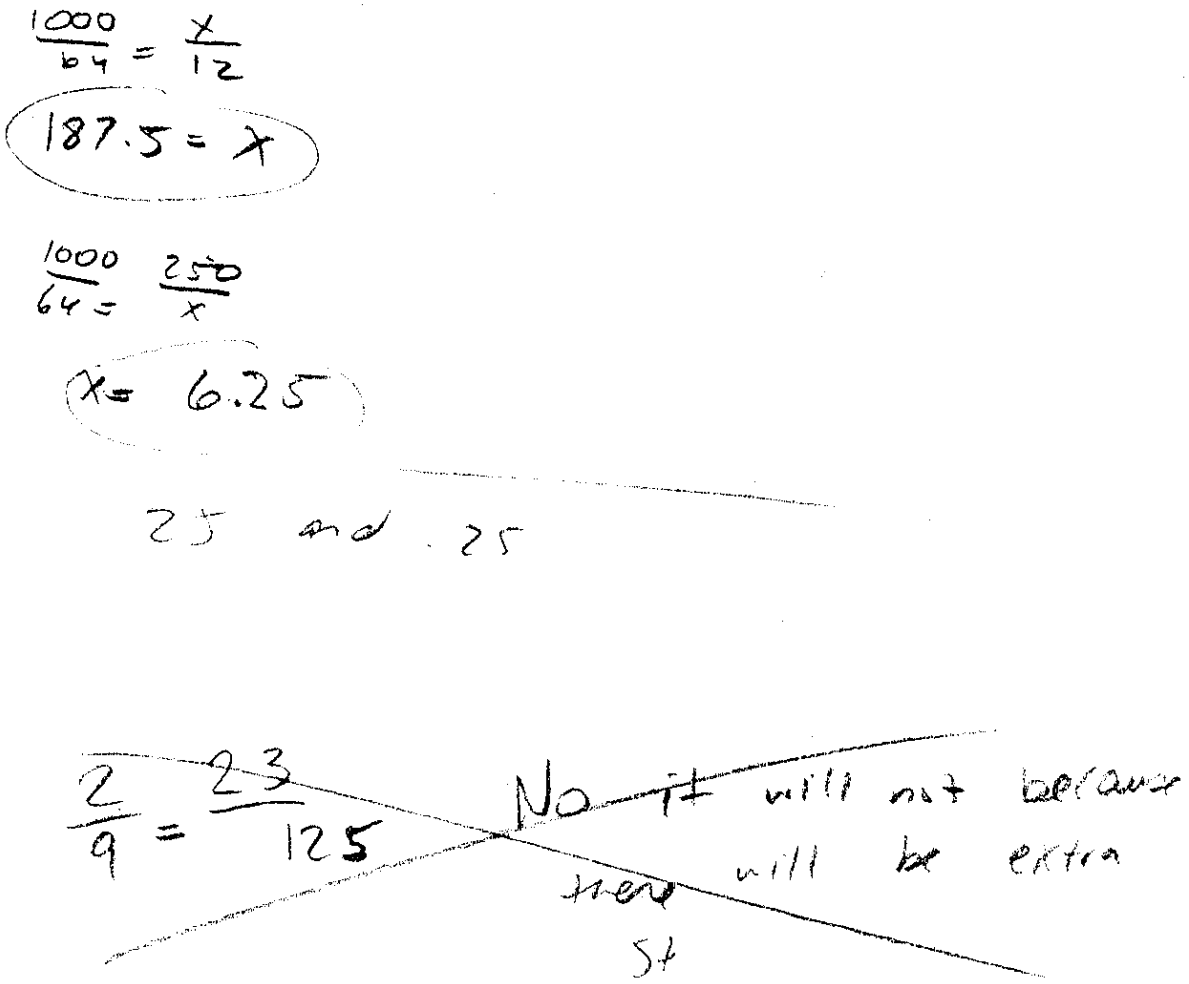
One set of possible dimentions for the rectangular region



Score Point: 1

The response to the first bullet is flawed—not a whole number. The response to the second bullet is incorrect; however, the third bullet is correct based upon an incorrect response to the second bullet.

MATHEMATICS
OPEN-ENDED RESPONSE



Score Point: 1

The response demonstrates a limited understanding of the concepts presented. The student has the correct proportion for the first bullet but does not round the answer to a whole number. The response to the second bullet is incorrect. The response to the third bullet is also incorrect.

MATHEMATICS
OPEN-ENDED RESPONSE

To get this answer I proportionalized the square to the number of cats.

$\frac{l}{64} = \frac{4}{1000}$ $\quad 625$ $\quad \frac{w}{64} = \frac{3}{1000}$ $\quad 46.875$

Approximately 110 points would be expected to fall inside the shaded rectangle.

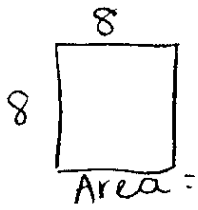
- $\frac{64}{1000} = \frac{16}{250}$

The area would approx. be 16 cm^2.

- 25 cm × 10 cm

Score Point: 1
The response has incorrect answers to the first and third bullets. The response contains a correct answer with work for the second bullet.

MATHEMATICS
OPEN-ENDED RESPONSE



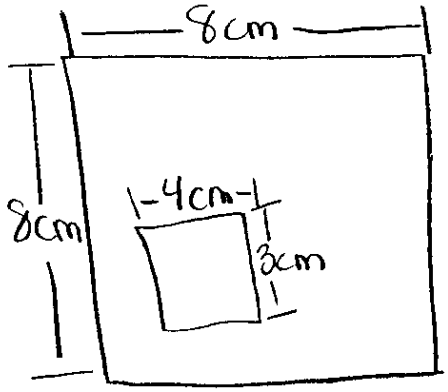
Area = 12

4096cm = 1,000pts

$$\begin{array}{r} 4096\text{cm} \\ -\ \ 12\text{cm} \\ \hline 4084 \end{array}$$

Score Point: 0
The response shows insufficient understanding of the problem's essential mathematical concepts. The response contains major errors.

MATHEMATICS
OPEN-ENDED RESPONSE

- 

$1000 \div 12 \approx 84$

So about 84 points would hit inside the little square.

$A = lw$ $A = 8(8)$ $A = 64$

$A = lw$ $A = 4(3)$ $A = 12$ 12 shaded area

$64 - 12$

52 nonshaded

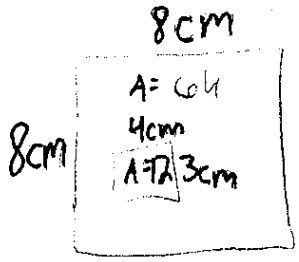
- $250 - 84 = 166$

So the square region would probably be a 5×5 square $25\ cm^2$.

Score Point: 0

The response shows insufficient understanding of the problem's essential mathematical concepts. The response contains major errors.

MATHEMATICS
OPEN-ENDED RESPONSE



Area of the 8x8 is 64.
Area of the 4x3 is 12.

Approximately 52 points would be expected to fall inside the shaded region.

Score Point: 0
The response shows insufficient understanding of the problem's essential mathematical concepts.
The response contains major errors.